

Discrete Math

The question is "what is fair"

The problem of how to divide something "fairly" is an ancient problem. In the Bible, King Solomon is asked to adjudicate between two women who both claim to be mothers of the same baby. You might recall that the solution proposed by Solomon was to divide the baby in half, and to give half to each mother.

There are many examples of things that need to be fairly divided. These examples might include dividing a pizza among 2 or more people, dividing the total pay between three people who do a job, Distributing the net proceeds from a company among the employees, dividing up the items in an estate amongst the heirs, allocating congressional seats in congress to states, and many other examples. Remember, as in the Solomon example, it is not always prudent to take out a knife.

Your first task then is to consider a plan to divide up three different items. Write a short paragraph describing how you can fairly adjudicate each situation.

- 1) Divide a large Papa Johns pepperoni pizza among three hungry people.

- 2) Two children find a stray dog in poor health that no one claims. The two children nurse the dog back to health. Both want to keep the dog and each family agrees that they can keep him. How do you decide which child gets to keep the dog (assume neither family wants to share the dog.)

- 3) Three children inherit an estate consisting of a number of items that are easily divided up and three items that each heir wants: the family house in Buckhead, a vacation house in Vail, and a yacht. How do you decide who gets what.

Now, carefully try to explain what the word "fair" means in the context of dividing something "fairly". Remember your objective is to define fairness in such way as to apply to the three situations above.

2.2 – Estate Division

Three members of a family are equal heirs to an estate. The estate consists of four major items of value and all members of the family decide that the items are worth more if possessed by a member of the family than they would be if sold on the open market. However, they decide to determine the market value of each item to have an idea how much to bid. The items are a family house in Vinings valued at \$400,000, a beach condo valued at \$190,000, a 36 foot sailboat valued at \$120,000, and a painting valued at \$50,000. In addition, the estate has \$350,000 in cash assets that will be split equally amongst the heirs.

All three heirs grew up in the family house and have settled in the Atlanta area with good jobs and incomes. They all would like to own one or more of the items above, and each realize that one or more of the items are worth a good deal more to them personally than the market value of the item.

1) Each heir submits a bid for each non-cash item in the estate. Assume these are the bids:

| | Bob | Frank | Joe |
|----------|-----|-------|-----|
| House | 420 | 440 | 400 |
| Condo | 200 | 180 | 225 |
| Boat | 100 | 120 | 90 |
| Painting | 75 | 60 | 60 |

2) The fair share is determined for each heir by finding the sum of his or her bids and cash and dividing this sum by the number of heirs.

| | Bob | Frank | Joe |
|----------------------|-----|-------|-----|
| Sum of bids and cash | | | |
| Share of estate | | | |
| Fair share | | | |

3) Each item in the estate is given to the heir who bid highest on that item. In case of a tie, the person who has won the least receives the item, or if still tied, a coin is tossed.

| | Bob | Frank | Joe |
|-----------------|-----|-------|-----|
| Objects awarded | | | |
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- 4) Each heir is given an amount of cash from the estate that is equal to his or her fair share (from step 2) less the amount that the heir bid on the objects he or she received. If this amount is negative, the heir pays this amount to the estate.

| | Bob | Frank | Joe |
|---------------|-----|-------|-----|
| Objects value | | | |
| Cash received | | | |

- 5) The remaining cash in the estate is divided equally among the heirs.
(To make our calculations easier, we will require that all bids be in thousands of dollars)

| | Bob | Frank | Joe |
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| Extra cash | | | |
| Total cash | | | |
| Final Settlement | | | |

1. Amy, Bart, and Carol are equal heirs to an estate containing a painting, a motorcycle, two tickets to the Redskins game against the Cowboys and \$15000. Their bids are shown below. Use the fair division algorithm to divide the estate fairly.

| | Amy | Bart | Carol |
|----------------------|------|------|-------|
| Painting | 2000 | 5000 | 3000 |
| Motorcycle | 4000 | 2000 | 3000 |
| Tickets | 500 | 100 | 300 |
| Sum of bids and cash | | | |
| Share of estate | | | |
| Fair share | | | |
| Objects awarded | | | |
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| Objects value | | | |
| Cash received | | | |
| Bank | | | |
| Extra cash | | | |
| Total cash | | | |
| Final Settlement | | | |

2. There are four equal heirs to an estate containing the items listed below and \$2000. Their bids are shown below. Use the fair division algorithm to divide the estate fairly.

| | Abby | Brian | Chuck | Dana |
|----------------------|------|-------|-------|------|
| Motorcycle | 2000 | 5000 | 4000 | 3000 |
| Jet Ski | 2500 | 3000 | 1000 | 2400 |
| Computer | 800 | 500 | 400 | 600 |
| Stereo | 1500 | 500 | 2000 | 1600 |
| Sum of bids and cash | | | | |
| Share of estate | | | | |
| Fair share | | | | |
| Objects awarded | | | | |
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| Objects value | | | | |
| Cash received | | | | |
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| Extra cash | | | | |
| Total cash | | | | |
| Final Settlement | | | | |

3. There are four equal heirs to an estate containing the items listed below and \$400,000. Their bids are shown below in thousands of dollars. Use the fair division algorithm to divide the estate fairly.

| | Kim | Riley | Jack | Chris |
|----------------------|-----|-------|------|-------|
| House | 260 | 290 | 260 | 280 |
| Beach House | 140 | 130 | 128 | 132 |
| Edsel | 44 | 40 | 48 | 44 |
| BB Cards | 72 | 82 | 68 | 76 |
| Prints | 100 | 90 | 80 | 104 |
| Sum of bids and cash | | | | |
| Share of estate | | | | |
| Fair share | | | | |
| Objects awarded | | | | |
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| Objects value | | | | |
| Cash received | | | | |
| Bank | | | | |
| Extra cash | | | | |
| Total cash | | | | |
| Final Settlement | | | | |

4. There are four equal heirs to an estate containing the items listed below. Their bids are shown below in thousands of dollars. Use the fair division algorithm to divide the estate fairly.

| | Lake House | Car | Boat |
|----------------------|------------|-----|------|
| Abby | 100 | 30 | 40 |
| Beth | 122 | 28 | 26 |
| Chrissy | 110 | 31 | 32 |
| Sum of bids and cash | | | |
| Share of estate | | | |
| Fair share | | | |
| Objects awarded | | | |
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| Objects value | | | |
| Cash received | | | |
| Bank | | | |
| Extra cash | | | |
| Total cash | | | |
| Final Settlement | | | |

5. Dana and Tom are retiring and selling their law firm. They will split the proceeds from this sale evenly. Both, however, want the firm's reserved seats at the local baseball stadium. Dana bids \$1200 for the seats and Tom bids \$900.

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| Sum of bids and cash | | |
| Share of estate | | |
| Fair share | | |

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| Objects awarded | | |
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| Objects value | | |
| Cash received | | |

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| Bank | | |
| Extra cash | | |
| Total cash | | |

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| Final Settlement | | |
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6. In the absence of a will, Evan, Franchesca, and Garth must fairly divide their uncle's estate consisting of a beach cabin and a boat. The group agrees to use the method of sealed bids.

| | Evan | Franchesca | Garth |
|-------|----------|------------|----------|
| Boat | \$16,000 | \$20,000 | \$12,000 |
| Cabin | \$80,000 | \$70,000 | \$90,000 |

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| Sum of bids and cash | | | |
| Share of estate | | | |
| Fair share | | | |
| Objects awarded | | | |
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| Objects value | | | |
| Cash received | | | |
| Bank | | | |
| Extra cash | | | |
| Total cash | | | |
| Final Settlement | | | |

7. Upon her death, a mother leaves a farm, townhouse, and 20 acres of timberland to her four children. Because other items in the estate had previously been distributed, each of the four heirs does not receive the same share of the estate. Ruth receives 30%, Steve 15%, Tina 35%, and Vern 20% of the estate. The four heirs submit sealed bids with the values they place on the properties.

| | Ruth | Steve | Tina | Vern |
|------------|-----------|-----------|-----------|-----------|
| Farm | \$380,000 | \$350,000 | \$325,000 | \$400,000 |
| Townhouse | \$250,000 | \$260,000 | \$300,000 | \$280,000 |
| Timberland | \$100,000 | \$140,000 | \$125,000 | \$130,000 |

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| Sum of bids and cash | | | | |
| Share of estate | | | | |
| Fair share | | | | |
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| Objects value | | | | |
| Cash received | | | | |
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| Final Settlement | | | | |

8. Art, Barbara, Carl, and Denise are heirs to an estate consisting of four cars (a '57 Chevy Impala, a '54 Ford pickup, a '62 Mustang convertible, and a '60 VW bug) and \$7500 in cash. Without collusion, each person submits sealed bids on the cars.

| | Art | Barbara | Carl | Denise |
|-------------------------|------|---------|------|--------|
| '57 Chevy Impala | 3500 | 7000 | 6000 | 3000 |
| '54 Ford Pickup | 2000 | 5500 | 4500 | 2500 |
| '62 Mustang Convertible | 5000 | 4000 | 4200 | 4500 |
| '60 VW Bug | 4000 | 5000 | 4800 | 5500 |

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| Sum of bids and cash | | | | |
| Share of estate | | | | |
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| Final Settlement | | | | |

9. Jose, Maria, and Eva will divide an estate consisting of a mountain cabin, a car, a painting, and \$20,000 in cash. Jose gets 50%, Maria gets 30% and Eva gets only 20% of the estate. They bid as follows:

| | Cabin | Car | Painting |
|-------|----------|--------|----------|
| Jose | \$60,000 | \$6000 | \$2000 |
| Maria | 52,000 | 4500 | 2000 |
| Eva | 61,000 | 5000 | 2500 |

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| Sum of bids and cash | | | |
| Share of estate | | | |
| Fair share | | | |
| Objects awarded | | | |
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| Cash received | | | |
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| Final Settlement | | | |

10. André, Bernardo, and Chantal have inherited part of their aunt's estate: a grandfather clock, a rolltop desk, a china cabinet, and \$3000. The group decides to use the method of sealed bids to fairly divide the inheritance and submits the following bids:

| | André | Bernardo | Chantal |
|---------|-------|----------|---------|
| Clock | 2500 | 3000 | 2800 |
| Desk | 3200 | 3500 | 3000 |
| Cabinet | 1000 | 2000 | 2500 |

If Bernardo is to receive 40% of the inheritance and André and Chantal each 30%, determine each person's fair share, the items received, and the amount of cash paid or received.

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| Sum of bids and cash | | | |
| Share of estate | | | |
| Fair share | | | |
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| Final Settlement | | | |

Estate Division Problem _____

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Sum of bids and cash

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Share of estate

Fair share

Objects awarded

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Objects value

Cash received

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Extra cash

Total cash

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Final Settlement

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Estate Division Problem _____

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Sum of bids and cash

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Share of estate

Fair share

Objects awarded

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Objects value

Cash received

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Extra cash

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Final Settlement

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Estate Division Problem _____

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Sum of bids and cash

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Share of estate

Fair share

Objects awarded

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Objects value

Cash received

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Final Settlement

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Estate Division Problem _____

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Sum of bids and cash

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Share of estate

Fair share

Objects awarded

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Final Settlement

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Estate Division Problem _____

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Sum of bids and cash

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Share of estate

Fair share

Objects awarded

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Objects value

Cash received

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Extra cash

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Final Settlement

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Estate Division Problem _____

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Sum of bids and cash

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Share of estate

Fair share

Objects awarded

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Objects value

Cash received

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bank

Extra cash

Total cash

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Final Settlement

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